A Plasma Controlled Directional Coupler*

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Summary—**A new type of waveguide directional coupler is described which has a discharge tube as the coupling element in the common narrow wall between the guides. The use of the discharge tube allows continuous control of the amount of coupling between the guides. The theory of the coupler is given with curves for designing such a coupler. The paper describes the results obtained from a 3-db coupler. Measurements were made at a frequency of 450 Mc, of VSWR, noise output, and switching speed. These show a VSWR of the order of 1.3 over the control range, and an excess noise temperature with a peak of 20,000°K at one value of control current. The coupler is capable of switching up to speeds of 1000 cps.**

Introduction

A WAVEGUIDE directional coupler is formed when two guides are coupled in such a manner that a traveling wave in one guide, excites in the other a wave propagated in one direction only. The performance of the coupler is specified in terms of the magnitude of coupling between the guides and the measure by which the wave in the secondary guide is unidirectional.

A single hole[1] coupler can be formed, by placing two guides at a specified angle with their broad walls in contact, and the hole placed in the region of contact. This arrangement has limited bandwidth over which directivity is maintained and only small amounts of coupling can be achieved. Tight coupling, though with limited bandwidth, can be obtained through the common broad wall by aligning the guides in the same direction and coupling them through a long slot whose length is of the order of half a wavelength. Broadbanding and tight coupling can be achieved by the use of multiple slots in the broad wall and one very effective coupler of this type employs a large number of transverse slots (serrated wall) in the broad wall.[2] This type of coupler has the advantage of the compactness which can be achieved since the narrow dimension of the guides can be reduced to a relatively small value without affecting the performance of the coupler except in its power handling capacity.

Directive coupling between the guides can also be effected by placing the guides so that they have a common narrow wall in which are a number of holes. A pair of slots in the common wall will give a limited range of coupling and a limited bandwidth. The amount of coupling and the bandwidth can be increased by increasing the number of slots. Very large couplings, up to 100 per cent, with high directivity over a wide bandwidth have been achieved using a long shaped slot in the common wall, across which lies a grid of wires.[3]

The directional couplers discussed so far have had fixed amounts of coupling.

In 1953, Tomiyasu and Cohn[4] described a new type of directional coupler capable of transferring variable amounts of microwave power, ranging from almost zero to one hundred per cent, from one waveguide to another. The coupling element was a long slot containing a grid of wires in the common narrow wall of a pair of rectangular waveguides. The degree of coupling was regulated by varying the physical length of the coupling slot by means of a mechanical shutter arrangement.

Damon[5] in 1955 demonstrated that the use of a ferrite post as a coupling element between two guides would give control of the amount of coupling, the control being effected by variation in the magnitude of an applied magnetic field. Other directional couplers using ferrites to give electrical control of the coupling were discussed by Berk and Strumwasser,[6] and Fox.[7]

In 1956, Pringle and Bradley[8] suggested that the Tomiyasu and Cohn coupler could be modified to act as a waveguide switch by filling the coupler with a suitable gas and inserting a cathode in the side arm. A discharge could then be created which would cut off propagation in the side arm.

It is evident that a thin plasma slab might be used in place of the Cohn-Tomiyasu grid of wires in the common narrow wall. Control of the plasma density by means of the current in the plasma tube would provide a variable effective electrical length of the slot, and hence, control of the division of power output from the waveguides. The mechanical control of the earlier device would thus be replaced by an electrical control with consequent advantage in ease and speed of operation. The theoretical aspects of this suggestion were investi-

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[1] R. F. Schwartz, "Bibliography on directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-2, pp. 58–63; July, 1954.

[2] J. A. Barksow, "Coupling of rectangular waveguides having a common broad wall which contains uniform transverse slots," 1957 IRE WESCON Convention Record, pt. 1, pp. 30–38.

[3] S. F. Miller and W. W. Mumford, "Multielement directional couplers," Proc. IRE, vol 40, pp. 1071–1078; September, 1952.

[4] D. Tomiyasu and S. B. Cohn, "The Transvar directional coupler," Proc. IRE, vol. 41, pp. 922–926; July, 1953.

[5] R. W. Damon, "Magnetically controlled microwave directional couplers," *J. Appl. Phys.*, vol. 26, pp. 1281–1282; October, 1955.

[6] A. D. Berk, and E. Strumwasser, "Ferrite directional couplers," Proc. IRE, vol. 44, pp. 1439–1445; October, 1956.

[7] A. G. Fox, "Notes on microwave ferromagnetics research," *Proc. IEE*, vol. 104, pt. B, suppl. 6, pp. 371–378; October–November, 1957.

[8] D. H. Pringle and E. M. Bradley, "Some new microwave valves employing the negative glow discharge," *J. Electronics*, vol. 1, ser. 1, pp. 389–404; June, 1956.

gated by Molik[9] and the results of his investigation showed that such a device was feasible. This present paper describes a directional coupler operating on this principle and presents the results of an experimental investigation into its behavior and properties.

THEORY

The mechanism of the power transfer between the waveguides can be described by assuming the superposition of two propagating modes which satisfy the boundary conditions in the coupling region. One mode is found to be symmetrical and the other antisymmetrical relative to the common wall. The two modes differ in phase velocity. The antisymmetrical mode is practically unaffected by the coupling element, since it has near-zero electric field in the slot and is propagated with a phase velocity essentially equal to the phase velocity in a single unperturbed guide. The symmetrical mode, however, is loaded by the plasma and propagates with a phase velocity affected by the plasma density. Since the phase velocities (and hence, the guide wavelengths) of the two modes differ, a superposition of the two modes results at one point in an electric field maximum in the main guide (at the point where the two modes are in phase), and a null in the secondary guide (where they are in phase opposition). At some other point along the guide the superposition of the modes will result in a reversal of the situation in which the field in the main guide is zero while that in the secondary guide is a maximum. The coupling length for which full power transfer occurs is equal to the distance between a maximum and an adjacent null in one waveguide. The coupling length is thus related in a relatively simple manner to the coupled-mode guide wavelengths.

The determination of the slot length required for full power transfer, and the way the power is divided between the guides for various values of plasma density, requires a determination of the propagation constant for the symmetrical and antisymmetrical modes. The problem of a rectangular waveguide with a slab of dielectric material at the center (Fig. 1), has been discussed by Collin[10] who shows that the solution to this problem involves a hydrid waveguide mode which has no component of electric field normal to the dielectric interface. The electric field thus lies in the longitudinal interface plane and the mode is referred to as a longitudinal-section-electric (LSE) mode. The eigenvalue equations for these modes are derived in Appendix I as

$$l \tan (hd) = - h \tan (lt/2) \quad (1)$$

$$h \cot (hd) = l \tan (lt/2) \quad (2)$$

[9] R. E. Molik, "Plasma Controlled Direction Couplers," M.S. thesis, Dept. of Engrg., University of California, Los Angeles; August, 1961.

[10] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 224–228; 1960.

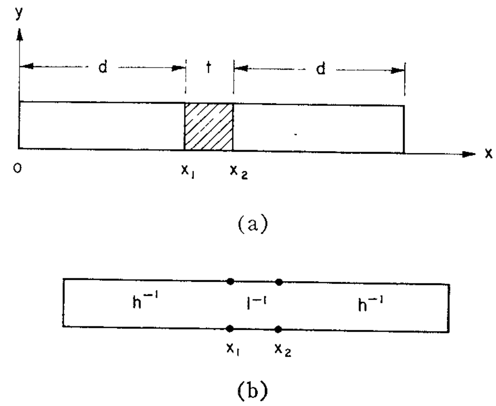


Fig. 1—(a) Cross section of direction coupler. (b) Equivalent transmission line circuit.

where t is the thickness of the plasma, and d is the width of one guide.

In addition there is the further relation

$$h^2 = l^2 + (1 - \kappa)\beta_0^2 \quad (3)$$

between the wave numbers h and l. In this expression κ is the relative dielectric constant of the plasma and

$$\beta_0 = \omega^2 \mu_0 \epsilon_0.$$

If we assume a loss free plasma then[11]

$$\kappa = [1 - (\omega_p/\omega)^2]$$

where

$$\omega_p^2 = \frac{Ne^2}{\epsilon_0 m}$$

$$\omega_p^2 = \frac{\rho}{\epsilon_0} \cdot \frac{e}{m};$$

$$\rho = \text{volume charge density} = eN;$$

and making this substitution in (3) gives

$$h^2 = l^2 + (\omega_p/\omega)^2 \beta_0^2. \quad (4)$$

Since the thickness of the plasma tube is small we can simplify (1) and (2) to

$$\frac{\tan (hd)}{h} = - t/2 \quad (5)$$

and

$$\frac{\tan (hd)}{h} = \frac{2}{t[h^2 - (\omega_p/\omega)^2 \beta_0^2]} \quad (6)$$

[11] R. F. Whitmer, "Principles of mirowave interaction with an ionized media," *Microwave J.*, vol., 2, pp. 17–19; February, 1959.

where l has been eliminated from (2) by the use of (4) to give (6). The axial phase constant is then given by

$$\beta = \sqrt{\beta_0^2 - h^2} \tag{7}$$

for the case where the fundamental modes only are considered.

Eq. (5) gives the value of h for the antisymmetrical mode and (6) the value of h for the symmetrical mode.

The full power transfer coupling length L for a pair of coupled propagating modes is given by

$$L = \frac{\pi}{\beta_s - \beta_a} \tag{8}$$

where β_s and β_a are the phase constants of the symmetrical and the antisymmetrical modes, respectively.

Solutions to (6) may be obtained from the intersections of plots of $m\tan\theta$ and $\theta/(\theta^2-n^2)$ where $m=t/2d$, $\theta=hd$ and $n=(\omega_p/\omega)(\beta_0 d)$. A limited but useful range of these curves is given in Fig. 2. The way in which the power is split between the two guides for a particular length of slot may then be calculated from

$$P_{1,2} = \tfrac{1}{2}(1 \pm \cos \pi L_1/L) \tag{9}$$

where L_1 is the length of the slot.

Experimental Equipment

The experimental coupler, Fig. 3(a) was designed to operate at a frequency of 450 Mc, the width of each guide being 21 inches while the narrow dimension was 1 inch. The coupler was constructed of $\frac{1}{8}$-inch aluminum sheets spaced 1 inch apart by aluminum bars of a square 1 inch cross section. The over-all length of the coupler was 8 feet. The rather shallow dimension of the guide was chosen as convenient from consideration of the plasma tube dimensions. The plasma tube, Fig. 3(b), was constructed of a circular cylinder of quartz 1 inch ID and 40 inches long. The tube fitted into a slot formed by removing an equal length of the 1-inch-thick center wall of the guide. The tube length was such that with zero current in the tube there was a half power split between the guides. The tube contained mercury vapor and had a cathode at one end which protruded through the bottom of the coupler. There was an auxiliary anode directly over the cathode and the main anode was at the far end of the tube.

Considerable difficulty was experienced in terminating the coupler because of the narrow dimension of the guide. A satisfactory solution was found by capacitively coupling the probe into the guide from the coaxial input cable by a large disk 2 inches in diameter and elevated $\frac{1}{8}$ inch from the broad face opposite the connector. The probe was located a quarter of a wavelength from a short circuit which closed the end of the guide. In addition it was necessary to have a tunable stub mounted close to the coaxial connector. In order to avoid mechanical distortion of the coupler and consequent mistuning of the termination the entire assembly was mounted on a rigid framework. The central dividing bar was tapered over the last six inches toward the tube to reduce undesirable reflections.

Prior to inserting the tube, the slot was sealed off and the terminating adjusted for minimum VSWR. A VSWR of 1.1 was achieved at 450 Mc.

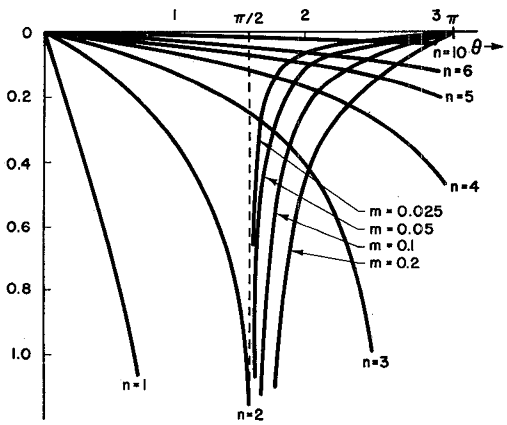
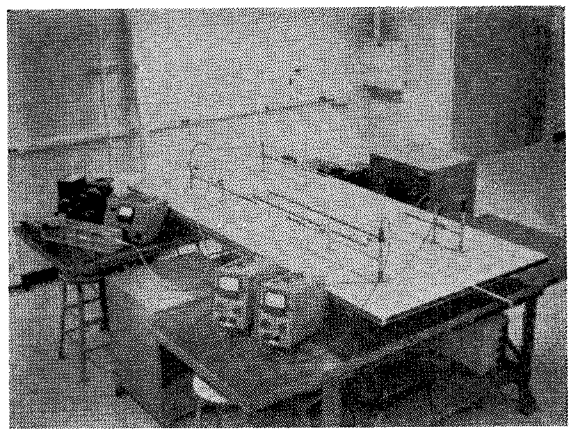
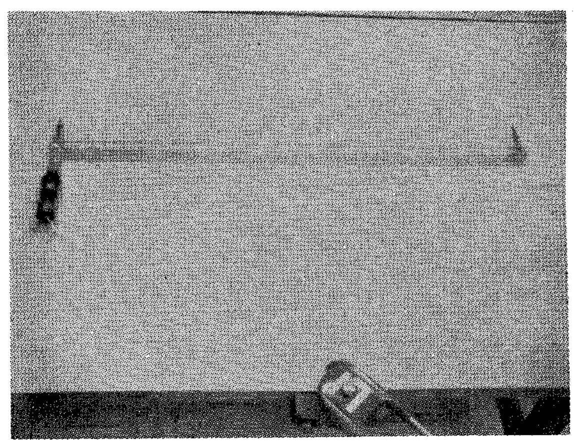


Fig. 2—Curves of $m \tan \theta$ and $\theta/(\theta^2 - n^2)$.



(a)



(b)

Fig. 3—Photograph of direction coupler and plasma tube.

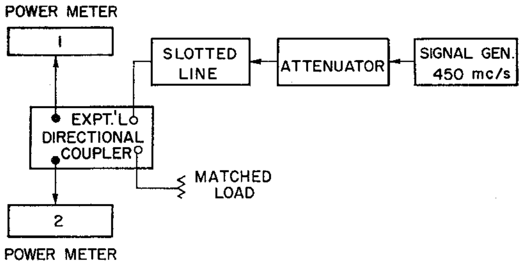


Fig. 4—Block diagram of experimental arrangement to measure control curves.

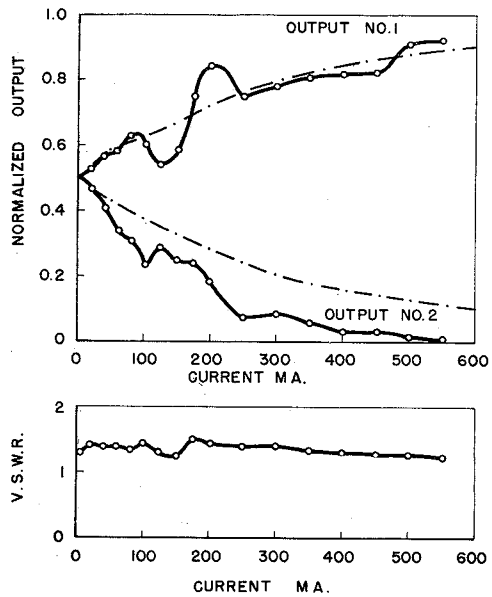
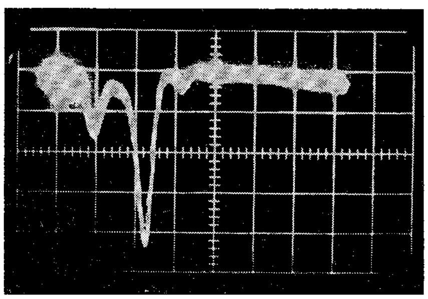


Fig. 5—Output of waveguides as a function of current in plasma tube.



current →

Fig. 6—Oscillogram showing Tonks-Dattner dips.

Results

Control Curves

The control curves for the coupler were obtained as shown in Fig. 4, by energizing one guide and measuring the two outputs at the distant end of the coupler. The fourth port was terminated in a matched load. The output power from both guides was measured as the current in the plasma tube was varied. Fig. 5 shows the results of this experiment. The outputs were equal at zero current, output number 1, the output of the guide to which the signal generator was connected, increasing to about 90 per cent as the current in the plasma tube was increased to 500 ma, while output number 2 decreased to a small value at the same current. The dotted curves are derived from the theory.

In order to correlate the experimental results with the theoretical curves, which have electron density as the independent variable, an experiment was performed in which the plasma tube was placed between an antenna radiating a 450-Mc signal and a detector. The absorption pattern was observed as the current in the plasma tube was varied. The pattern (Fig. 6) showed the Tonks[12] Dattner[13] dips with three clearly defined dips at 70, 140 and 200 ma. At the main dip of 140 ma

$$\left(\frac{\omega_p}{\omega}\right) = (1 + K_{\text{eff}})^{1/2} \tag{10}$$

where K_{eff} allows for the fact that the plasma is confined in a cylindrical glass tube of finite thickness. [Eq. (10) is derived in Appendix II.] For the dimensions of this particular tube, K_{eff} has the value 1.53. Using this value, the theoretical curves were plotted alongside the experimental curves. It will be seen there is fair agreement between the theoretical and the experimental results in spite of the fact that the theory is based on a rectangular slab of plasma rather than the cylindrical section used, and that the electron density was not uniform over the cross section of the tube. If a rectangular section had been used there would have been some difficulty in assigning a value to the electron density-current relation at resonance. In addition there would have been the difficulty of construction of such a tube. It will be observed that the plasma resonances have a pronounced effect on the control curves of the device. The main resonance, particularly, produced a violent dip in the direct output. In spite of this behavior the input VSWR did not rise beyond 1.5 over the whole range of control current.

Deionization Time

The speed of operation of the coupler is determined by the deionization time of the gas in the tube. In order to obtain an estimate of this time an experiment was carried out in switching the current in the plasma tube between 500 ma and 10 ma. The switching was accomplished by means of a mercury switch short circuiting a resistor in the lead to the main anode at 60 cps. The deionization time was thus found to be of the order of 60 μsec, which would allow switching speeds up to at least 1000 cps. Gases other than mercury vapor were not experimented with, but it is to be expected that the switching speed could be increased by using a lighter gas than mercury vapor.

[12] L. Tonks, "The high frequency behaviour of a plasma," *Phys. Rev.*, vol. 37, pp. 1458–1483; June 1, 1931.

[13] A. Dattner, "The plasma resonator," *Ericcson Technics* (Stockholm), vol. 13, no. 2, pp. 310–350; 1957.

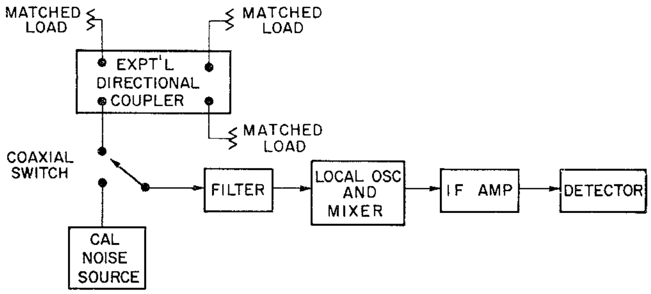


Fig. 7—Block diagram of experimental arrangement for noise measurement.

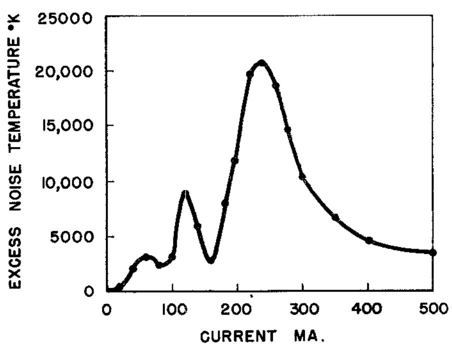


Fig. 8—Excess noise temperature as a function of current in plasma tube.

Noise

A measurement of the noise output of the coupler for various values of the plasma current was made by sampling a 2-Mc bandwidth of the noise. The system was calibrated with a standard noise tube source. The circuit is shown in Fig 7 and the results in Fig 8. The curve showed three peaks, the largest noise output occurring at the peak at the highest current. The excess noise temperature at this current was 23,000°K and thereafter dropped rapidly down to 3500°K as the current was increased beyond this value. Such high temperatures are of course to be expected when plasma tubes are used near resonance and particularly where the action of the device employing the tube depends on tight coupling between the tube and the guide.

Conclusions

The plasma controlled directional coupler which has been investigated controls the division of power between two guides. The length of the plasma coupling tube was such that the maximum coupling corresponded to 3 db of coupling. By increasing the length of the tube the maximum coupling could be raised to 100 per cent. This type of coupler has the important advantage of being controlled electrically either by variation of a direct current to give continuous control or if desired by a pulse or switching circuit. The control characteristic could be employed to maintain the amount of coupling at a desired level against changes caused by variation in signal frequency. The control curves of the device constructed were not such as to give smooth control due to resonance effects in the plasma tube. This would be immaterial in the case of a switching application if the switching range of the current were great enough to span the currents at which the resonances occurred.

In order to improve the shape of the control curves, further studies should be directed to reducing, or removing, the resonance effects to current levels where they will be outside the control range of the device. For example, referring to Fig. 2, for the coupler constructed the parameter m has the value 0.025, and $n/5$ gives the values of ω_p/ω. It will be seen that θ, the solution of

$$m \tan \theta = \theta/(\theta^2 - n^2)$$

shows appreciable variation only when ω_p/ω increases beyond unity, and hence, only for those values of ω_p/ω is there an appreciable change in the phase velocity of the symmetrical mode, and consequent change in the full power coupling length. On the other hand, if the width of the plasma tube is increased to give a value of $m=0.2$ (corresponding to a width of 8 inches in the present coupler), then the greater part of the variation of θ would take place in the region below $\omega_p/\omega=1$, and the major part of the range would lie below the major resonances. The control current would not be increased greatly in this wider tube since the increase in current required to maintain the current density over the increased cross section would be offset by the fact that the control range would correspond to a lower current density range.

A further advantage would accrue in that the noise output would be reduced since the resonances now lie outside the working range and the electron density is now so much less. Further the deionization time would not be greatly increased if the height of the tube were kept constant since the controlling factor is the time taken by the ions to diffuse to the walls of the tube.

The reduction in noise output would increase the dynamic signal range over which the coupler is useful. The range is limited at the low signal levels by the degradation of the signal-to-noise ratio by the noise generated in the device, while, at high power, an upper limit is set by the increase in plasma density caused by the incident microwave signal power. At very high power levels there would be a significant increase in plasma density which might be large enough to make control impossible.

The directional coupler investigated shows promise of being capable of development into a device with the desirable features of speed and ease of control. It is inherently a narrow-band device since the symmetrical and antisymmetrical modes are functions of frequency and a 3-db coupler, for example, will be too long or short at frequencies other than the designed frequency. Bandwidth will therefore depend on the tolerable decrease in power transfer over the frequency range. The limitations at the extremes of the power range should not be so severe in a well-designed coupler as to seriously restrict the dynamic range, though further study is required to establish its power handling capability and bandwidth.

In comparison with directional couplers employing ferrites to give variable electrical control of the coupling, these are also subject to high power limitations, since the ferrite absorbs microwave power and this gives rise to a subsequent increase in temperature and change in the parameters of the device. Speed of response is also restricted in such couplers by the necessity for changing the strength of the strong magnetic field which is used to control the parameters of the ferrite and hence the amount of coupling. The two devices may be considered complementary since the plasma coupler operates well at frequencies of several hundred megacycles while the design of a ferrite directional coupler for this frequency range may call for exceedingly large ferrite slabs and impractically large magnets. At high frequencies the plasma coupler may demand current densities which may be excessive for a practical device, while on the other hand devices employing ferrites are well suited for this range.

Appendix I

The eigenvalue equations may be obtained[10] by considering the equivalent transmission line circuit of the wavequide-plasma system in the transverse plane as shown in Fig. 1. By considering propagation to take place in the x direction, the equivalent circuit comprises three sections of transmission line with characteristic impedances proportional to the wave impedances (inversely proportional to the wave numbers) of the sections. The two sections representing the guides are short circuited at the ends. Referring to Fig. 1, at $x=x_2$ the input impedance is

$$jh^{-1}\tan(hd)$$

at $x=x_1$ the input impedance is

$$l^{-1}\frac{jh^{-1}\tan(hd)+jl^{-1}\tan(lt)}{l^{-1}-h^{-1}\tan(hd)\cdot\tan(lt)}.$$

At $x=x_1$ the input impedance looking to the right must be equal to the negative value of the input impedance seen looking to the left, hence we have

$$-jh^{-1}\tan(hd)=l^{-1}\frac{jh^{-1}\tan(hd)+jl^{-1}\tan(lt)}{l^{-1}-h^{-1}\tan(hd)\tan(lt)}$$

where t is the thickness of the plasma slab, and d is the width of one guide, h and l are the wave numbers associated with the guides and the plasma slab, respectively.

This equation simplifies to

$$h^2\tan(lt)+2hl\tan(hd)-l^2\tan(lt)\tan^2(hd)=0.$$

Solving for tan (hd) we get the eigenvalue equations

$$l\tan(hd)=-h\tan(lt/2)$$

and

$$h\cot(hd)=l\tan(lt/2).$$

Appendix II

Consider an infinite glass cylindrical tube of dielectric constant κ_g containing a uniform plasma immersed in a uniform electric field of strength E. The axis of the cylinder is perpendicular to the electric field.

The original potential outside the cylinder is of the form

$$V=Ex=Er\cos\theta$$

where the orientation of the axis is shown in Fig. 9.

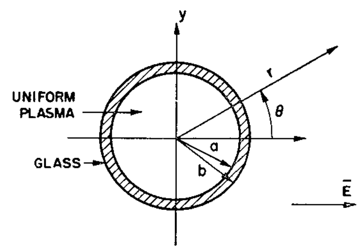


Fig. 9—Co-ordinate system used in calculation of K_{eff}.

At infinity the potential superimposed on this due to induced charges on the cylinder must vanish so that no terms of the form r^n can occur in it. Since this potential must be symmetrical about the x axis no term involving $\sin\theta$ can occur. The final potential outside must therefore be of the form

$$V_0=Er\cos\theta+\sum_{n=1}^{\infty}A_nr^{-n}\cos n\theta.$$

Since the values $r=0$ and $r=\infty$ are excluded from the dielectric both r^n and r^{-n} can occur but $\sin n\theta$ terms are thrown out as before.

The potential in the dielectric must therefore be of the form

$$V_g=\sum_{n=1}^{\infty}(B_nr^n+C_nr^{-n})\cos n\theta.$$

Inside the plasma there must be no terms in r^{-n} since the potential must be zero (or finite) at $r=0$. Again because of the symmetry there will be no terms in $\sin n\theta$. Thus the potential in the plasma must be of the form

$$V_p=\sum_{n=1}^{\infty}D_nr^n\cos n\theta.$$

The coefficients A_n, B_n, C_n and D_n may now be evaluated by satisfying the boundary conditions at the air-glass and glass-plasma boundaries. These are

$$\text{At} \quad r = b \qquad \frac{\partial V_0}{\partial r} = \kappa_g \frac{\partial V_g}{\partial r} \quad \text{and} \quad V_0 = V_g$$

$$\text{At} \quad r = a \qquad \kappa_g \frac{\partial V_g}{\partial r} = \kappa_p \frac{\partial V_p}{\partial r} \quad \text{and} \quad V_p = V_g$$

where κ_g is the relative dielectric constant of the glass tube and κ_p the relative dielectric constant of the plasma. We are interested only in the potential in the plasma. Carrying out the process indicated above we find that

$$V_p = \left\{ \frac{-4E\kappa_g/a^2}{[(1+\kappa_g)(\kappa_p+\kappa_g)/a^2] - [(1-\kappa_g)(\kappa_p-\kappa_g)/b^2]} \right\} \cdot r \cos\theta.$$

The resonance condition occurs when the denominator is zero. From this and the fact that

$$\kappa_p = (1 - \omega_p^2/\omega^2)$$

we obtain

$$\left(\frac{\omega_p}{\omega}\right)^2 = 1 + \frac{\kappa_g\left[\frac{b^2}{a^2}(1+\kappa_g) - (\kappa_g - 1)\right]}{\frac{b^2}{a^2}(1+\kappa_g) + (\kappa_g - 1)}.$$

Thus

$$\kappa_{\text{eff}} = \kappa_g \frac{\frac{b^2}{a^2}(1+\kappa_g) - (\kappa_g - 1)}{\frac{b^2}{a^2}(\kappa_g + 1) + (\kappa_g - 1)}.$$

For the tube used, $a/b = 0.853$ and $\kappa_g = 3.78$ giving $\kappa_{\text{eff}} = 1.53$.

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Correspondence

Couplings in Direct-Coupled Waveguide Band-Pass Filters*

Couplings in direct-coupled waveguide band-pass filters can be described in two different ways. Cohn[1] describes couplings in terms of normalized susceptances of the coupling circuit elements (*e.g.*, posts or irises). Dishal[2] describes interstage couplings in terms of a coefficient of coupling between adjacent resonators. Although Cohn has briefly mentioned coefficients of coupling in his paper,[1] the interchangeability of the two methods of describing the couplings has not been clearly established.

The low-pass prototype of the direct-coupled waveguide band-pass filter is shown in Fig. 1.

$$K_{12} = \frac{\Delta f_{12}}{f_0} = \frac{1}{\omega_1'\sqrt{C_1 L_2}}\left(\frac{f_2 - f_1}{f_0}\right) \qquad (1)$$

where

K_{12} = coefficient of coupling between resonators 1 and 2
f_0 = center frequency of filter
ω_1' = pass band edge of low-pass prototype
f_2 and f_1 = corresponding pass band edges of waveguide filter
C_1 and L_2 = circuit elements in low-pass prototype.

Δf_{12} is a coupling bandwidth that can be measured by simple experimental techniques. The utilization and measurement of this coupling bandwidth is described in detail by Dishal.[3]

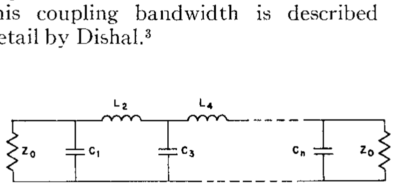

Fig. 1—Low-pass prototype.

The lumped-circuit band-pass equivalent of the direct-coupled waveguide band-pass filter is shown in Fig. 2. This equivalency is usually satisfactory for narrow-bandwidth filters (*i.e.*, $\leqq 5$ per cent bandwidth).

$$B_{12} = \frac{1 - \frac{L^2}{C_1 L_2}}{\frac{L}{\sqrt{C_1 L_2}}} \cong \frac{1}{\frac{L}{\sqrt{C_1 L_2}}}$$

for narrow-bandwidth filters (2)

where

$$B_{12} = \frac{Z_0}{2\pi f_0 L_{12}} = \text{normalized susceptance of coupling circuit element.}$$

$$L = \frac{\pi}{2\omega_1'}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2\left(\frac{f_2 - f_1}{f_0}\right)$$

= frequency variable (3)

λ_{g0} = guide wavelength at filter center frequency
λ_0 = free space wavelength at filter center frequency.

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